INTERMITTENCY AND SCALE SIMILARITY IN THE STRUCTURE OF A TURBULENT FLOW

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Intermittency is the nonuniform distribution of eddy formations in a stream. The modulus or the square of the vortex field, the energy dissipation velocity or related quantities quadratic in the gradients of velocity and temperature (of the concentration of passive admixture) may serve as indicators.

It is necessary to distinguish the intermittency in a sporadically turbulent stream (in particular near the border between a turbulent and nonturbulent region) from the intermittency in a developed turbulent stream. The cause of intermittency is the instability of eddy formations and in connection with this the random character of the process of breakdown of larger vortices into smaller ones.

The study of intermittency has a number of aspects. In the first place, there is the statistical analysis of fields of the stated type which characterize the nature of intermittency. Statistical features of the vortex field [1, 2] represent a basic interest for the general theory of turbulence. To date it is fully possible to carry out the necessary measurements [3].

Another aspect is the study of the influence of intermittency process (in terms of variation of dissipation) on the structure of the velocity and temperature fields, and in particular on the energy spectrum. The existence of such influence has been established by Landau [4] and developed in [5] and [6] (see also a somewhat different statement of the problem in [7]. Apparently, the effect of the influence of variation of dissipation on the energy spectrum is most important in the so far little explored field of motions involving very high wave numbers [7–9]. A universal form of the spectrum can be obtained, generally speaking, through the introduction of a geometrical averaged spectral density [1]: $\widetilde{E}(k) = \exp\{\langle \ln E(k) \rangle\}$, where $E(k)$ is stochastic spectral energy density (for a fixed value of dissipation). The angular brackets indicate averaging.

Connected with the study of intermittency, is, of course, the problem of the choice of the optimal averaging time for the different statistical characteristics of flow. This problem presents itself foremost in flows with a broad spectrum of motions, e. g. in atmospheric turbulence [5].

In the investigation of the characteristics of the intermittency process it is important above all to attempt to discern universal laws which would not depend upon the large-scale structures of the stream and, possibly, upon the Reynolds number (provided it is sufficiently large; the developed stage of turbulence is discussed below).

In the description of the intermittency process it has been found useful the notion of the breakdown coefficient, namely the ratio of values of the nonnegative field, averaged over different scales (Sect. 1). Using the breakdown coefficient, the spectrum and other field characteristics of the energy dissipation type were obtained [10–13]. In a defined range of scale the spectrum has been found to be proportional to $k^{-1+\mu}$ (k is the wave number $0 < \mu < 1$). This is in agreement with experimental data [14–20, 24] which give the value of the parameter $\mu \approx 0.4$. The notion of the breakdown coefficient was also utilized in [21] and (24] for the establishment of the logarithmic-normal law (proposed earlier [5, 6]) of the distribution of energy dissipation and for the deduction of an analogous two-dimensional distribution. The experimental data [22–25, 17, 18] for quantities of the type of the dissipation show distributions close to logarithmic normality, although there are systematic deviations which are treated in Sect. 5.

In the paper under consideration which represents a development of [13] it is shown that universal laws have significance for the statistical characteristics of the breakdown coefficient. As far as formulas for the spectrum and probability distributions of the initial fields are concerned,these formulas actually are obtained by means of an extrapolation, and therefore they are of an approximate and nonuniversal nature.

It is shown further that the probability distribution of the breakdown coefficient tends to logarithmic normality with increase of the scale ratio of the averaging process. This tendency, however, is sufficiently slow, determined by the logarithm of the scale ratio or by the logarithm of the corresponding Reynolds number. Moreover the moments of the breakdown coefficient do not tend towards the corresponding expressions derived from the limit logarithmic normal law (the situation is rather unique for processes which take place in nature). This latter observation is significant not only for the breakdown coefficients but also for all fields of the energy dissipation type. This is confirmed by experiments (Sect. 5). The state of the art of experimental research fully permits the direct determination of the statistical characteristics of the breakdown coefficient and by this very fact to correlate most closely theory and experiment.

It is noted that the paper of Kolmogorov [6] contains the defined hypotheses of similarity expressed in terms of a ratio of velocity differences. This ratio may go to infinity which renders the theoretical and experimental research a little more difficult. Nevertheless it is possible that universal laws exist only for relative characteristics of the turbulent flow of the kind to which the breakdown coefficient belongs.

1. Breakdown coefficient. Let us consider a nonnegative random function $y(x)$. This function may be represented in particular by one of the following quantities:

$$\left(\frac{\partial v_1}{\partial x}\right)^2, \quad \left(\frac{\partial v_{2,3}}{\partial x}\right)^2, \quad \Omega^2_{1,2,3}, \quad \left(\frac{\partial \theta}{\partial x}\right)^2 \tag{1.1}$$

considered as a function of the coordinate x in direction of the mean velocity of the stream. This is in accord with the interpretation with the aid of the "frozen flow"-hypothesis of experimental readings in time. Here v_1 and $v_{2,3}$ are, respectively, longitudinal and one of the transverse components of the velocity fluctuations, $\Omega_{1,2,3}$ is any component of eddy fluctuation, θ is the fluctuation of temperature (of concentration of a passive admixture). Taking account of the conditions of incompressibility and of local isotropy, the mean values of the quantities (1.1) are equal, respectively, to

$$\frac{1}{15}\frac{\langle\varepsilon\rangle}{\nu}, \quad \frac{2}{15}\frac{\langle\varepsilon\rangle}{\nu}, \quad \frac{1}{3}\frac{\langle\varepsilon\rangle}{\nu}, \quad \frac{1}{3}\frac{\langle N\rangle}{\varkappa} \tag{1.2}$$

Here $\langle\varepsilon\rangle$ is the mean value of the dissipation velocity of kinetic energy, $\langle N\rangle$ is the mean equalizing velocity of fluctuations of temperature (of concentration of a passive admixture), and ν and $\varkappa$ are the coefficients of kinematic viscosity and of thermal diffusivity, respectively.

The ratio of the values of the function $y(x)$, averaged over two sections

$$q_{r,l}(h,x) = y_r(x')/y_l(x), \qquad r < l \tag{1.3}$$

$$y_l(x) = \frac{1}{l}\int_{x-\frac{1}{2}l}^{x+\frac{1}{2}l} y(x_1)\,dx_1, \qquad -\frac{1}{2} \leqslant h = \frac{x'-x}{l-r} \leqslant \frac{1}{2}$$

will be called the breakdown coefficient. Restriction with respect to h signifies that the smaller section is included in the larger.

In accordance with the experimentally confirmed theory of local homogeneity and local isotropy of turbulence the functions (1.1) may be taken to be homogeneous and isotropic with scales smaller than the characteristic macro-scale L. For such scales the probability distributions of the quantity $q_{r,\,l}$ do not depend upon x; generally speaking, however, they depend upon $|h|$ (*). This dependence defines the nonhomogeneity of the breakdown.

As an example let us consider the homogeneous Markov sequence of nonnegative quantities y_k $(k = \ldots, 1, 2, 3, \ldots)$ with the density distribution of each of their $W(y)$ and with transition probability

$$P(y_{k+1} \mid y_k) = \alpha\delta(y_{k+1} - y_k) + (1-\alpha)W(y_{k+1}) \quad (0 < \alpha < 1) \tag{1.4}$$

where ($\delta(y)$ is the delta function. It is easy to show that $P^*(y_k \mid y_{k+1})$ is obtained from (1.4) by exchanging places of y_k and y_{k+1}, i.e. the sequence is isotropic. Let us denote

$$Q_m \equiv q_{1,3}(h_m, 2) = \frac{3y_m}{y_1+y_2+y_3} \qquad (m = 1, 2, 3;\ h_1 = -h_3 = -\,{}^1/_2,\ h_2 = 0)$$

Evidently

$$Q_1 + Q_2 + Q_3 = 3, \qquad \langle Q_1\rangle = \langle Q_3\rangle$$

Taking (1.4) into account, we may write

$$\langle Q_1\rangle - \langle Q_2\rangle = \frac{3}{2}\alpha(1-\alpha)\int_0^\infty\int_0^\infty \frac{(y-y')^2}{(y+2y')(y'+2y)} W(y)\,W(y')\,dy\,dy' \geqslant 0 \tag{1.5}$$

From this it is seen that breakdown is nonhomogeneous. The integral appearing in (1.5) vanishes only in the case of determinate y_k, when $W(y) = \delta(y-a)$. The degree of breakdown nonhomogeneity defined by the difference (1.5) does not exceed ${}^3/_{16}$ in the present example.

With respect to the quantities (1.1) it would be desirable to investigate experimentally the nonhomogeneity of breakdown, in the first place the dependence upon h of the mean value of the breakdown coefficient.

If the homogeneous random function $y(x)$ satisfies the ergodic conditions (it is sufficient that the correlation function vanishes at infinity in exponential fashion), then

$$y_l(x)_{l\to\infty} \to \langle y\rangle, \quad q_{r,l}(h,x)_{l\to\infty} \to y_r(x')/\langle y\rangle \tag{1.6}$$

Knowing the statistic characteristics of the breakdown coefficient, we can (with the aid of (1.6)) carry out the transition to the usual presentation. In particular

$$\langle y(x+r)\,y(x)\rangle\langle y\rangle^{-2} = \lim_{l\to\infty}\frac{1}{2}\frac{d^2}{dr^2}[r^2\langle q^2_{r,l}\rangle] \tag{1.7}$$

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*) It is evident that upon h there depends the joint probability distribution for the quantities $y_r(x')$ and $y_l(x)$ and, in particular, the correlation between these quantities.

Concluding this Section we give a simple yet important inequality, which follows from the nonnegative nature of $y(x)$

$$q_{r,l}(h, x) \leqslant l/r \tag{1.8}$$

2. Similarity of scale. Aside from the macroscale L for the function $y(x)$ there exists a microscale l_* defined by the viscosity (heat transfer, diffusion). The magnitude of l_* may differ from the Kolmogorov microscale $l_\nu = \nu^{3/4} \langle\varepsilon\rangle^{-1/4}$ (or its generalization in the case of passive admixture) by some degree of Reynolds number [10, 12]. If in the interval between L and l_* there are no other characteristic scales defining the behavior of the random function $y(x)$, then it is required that for $L \gg l > r \gg l_*$ the probability distribution for the breakdown coefficient (1.3) would depend only on the ratio l/r and on $|h|$.

Let us introduce the interval section of length ρ between r and l, with its center at the point $x'' = x + h(l - \rho)$. We have

$$q_{r,l}(h, x) = q_{r,\rho}(h, x + h(l - \rho))\, q_{\rho,l}(h, x) \tag{2.1}$$

All three breakdown coefficients appearing in (2.1) are governed by the same quantity h. We require that in the interval of scales defined above the factors on the right of Eq. (2.1) be statistically independent. Justifications for the use of this kind of condition were presented in [10, 21] where a somewhat specific scheme of breakdown of large cubes into small ones with the same decrease of scale for each of n successive steps has been investigated.

The justification in that case was based upon the fact that the breakdown should retain universality until the corresponding Reynolds number (defined by the energy dissipation averaged over the volume under consideration) is sufficiently large and the corresponding scales are large compared with l_*.

As will be seen below, the statistical independence of successive breakdown coefficients is necessary and sufficient in order that the moments retain their exponential character (the latter is confirmed experimentally). We shall call the two conditions formulated above (namely the dependence of the probability distribution for the breakdown coefficient solely upon the ratio of scales and h and the statistical independence of two successive coefficients with the same h) the conditions of similarity of scale, and the corresponding interval — the interval of similarity of scale.

Let us consider the moments of the breakdown coefficient

$$a_p(l/r, h) = \langle q_{r,l}^p(h, x)\rangle \tag{2.2}$$

Here p is positive (not necessarily integer) number. From the conditions of similarity of scale with (2.1) taken into account, we obtain

$$a_p(l/r, h) = a_p(\rho/r, h)\, a_p(l/\rho, h)$$

and, since ρ is arbitrary,

$$a_p(l/r, h) = (l/r)^{\mu_p(h)} \tag{2.3}$$

The probability distribution for the breakdown coefficient is concentrated in the finite interval (1.8), then it is easy to see that the following restrictions must be applied:

$$\mu_{p+\delta}(h) - \mu_p(h) \leqslant \delta \qquad (\delta \geqslant 0) \tag{2.4}$$

Since $\mu_0(h) \equiv 0$, we have

$$\mu_p(h) \leqslant p \tag{2.5}$$

If the nonhomogeneity of breakdown (dependence on h) is disregarded, we have, as shown in [13]

$$\mu_1 = 0, \quad 0 < \mu_2 \equiv \mu < 1 \tag{2.6}$$

and the experimental data evaluated in terms of the breakdown coefficient with the aid of (1.7) give $\mu \approx 0.4$. The inequality (2.5) is changed by a stronger one

$$\mu_p \leqslant \mu + p - 2 \qquad (p \geqslant 2)$$

Let us consider the series for the characteristic function of the breakdown coefficient

$$\varphi(s, l/r, h) = \langle \exp\{isq_{r,l}(h, x)\}\rangle = \sum_{p=0}^{\infty} \frac{(is)^p}{p!}\left(\frac{l}{r}\right)^{\mu_p(h)} \tag{2.7}$$

From the inequality (2.5) it follows that this series converges absolutely and uniformly in any finite interval $|s| \leqslant S$. A simple transformation of the series (2.7) with (2.1) and (2.3) taken into account yields

$$\varphi(s, l/r, h) = \langle \varphi(sq_{r,\rho}, l/\rho, h)\rangle = \langle \varphi(sq_{\rho,l}, \rho/r, h)\rangle$$

This signifies statistical independence of successive breakdown coefficients. Thus, the requirements of similarity of scale are proven to be not only sufficient but also necessary in order that the moments have an exponential character.

It is easy to verify that the inequality (2.5) ensures the fulfilment of the Carleman condition [26]

$$\sum_{p=1}^{\infty} (a_{2p})^{-\frac{1}{2p}} = \infty$$

which is sufficient for the probability distributions to be uniquely defined by moments of integer order. It is noted that the limiting logarithmic-normal distribution considered in the next Section does not have this property [26].

The set of exponents of power $\mu_p(h)$ $(p = 0, 1, 2...)$ uniquely fixes the probability distribution for the breakdown coefficient for any arbitrary value l/r. A general form of the distribution may be obtained by introducing the characteristic function for the logarithm of the breakdown coefficient

$$\psi(s, l/r, h) = \langle \exp\{isz_{r,l}(h, x)\}\rangle$$

$$z_{r,l}(h, x) = \ln q_{r,l}(h, x)$$

The conditions of similarity of scale with (2.1) taken into account yield

$$\psi(s, l/r, h) = \psi(s, \rho/r, h)\,\psi(s, l/\rho, h)$$

By virtue of the arbitrariness of ρ we obtain

$$\psi(s, l/r, h) = (l/r)^{-\alpha(s, h)} = \exp\{-\alpha(s, h)\ln(l/r)\} \tag{2.8}$$

The function of $\alpha(s, h)$ satisfying the condition of normalized total probability $\alpha(0, h) = 0$ has universal significance. This statement may be confirmed through a direct experimental check by measurement of the probability distribution or of particular moments of the breakdown coefficient for various values of l/r. For the exponents of the moments we have

$$\mu_p(h) = -\alpha(-ip, h) \tag{2.9}$$

If it is possible to disregard the effect of the nonhomogeneity of breakdown, then $\alpha(s)$ is a universal (complex) function of a single argument. The determination of this function is an important task for further experimental investigations of the structure of turbulent flow.

3. Ultimate probability distribution. For simplicity of writings, the dependence of all quantities on the parameter h will be omitted. The cumulants of the distribution of the logarithm of the breakdown coefficient are defined by the derivatives of the function $\alpha(s)$ at zero (provided these derivatives exist). All cumulants are proportional to the logarithm of the scale ratio

$$\varkappa_p\left(\frac{l}{r}\right) = (i)^{-p} \frac{d^p \ln \psi(s, l/r)}{ds^p}\bigg|_{s=0} = (i)^{2-p}\alpha^{(p)}(0)\ln\frac{l}{r} \tag{3.1}$$

Assuming that the mean value $(\varkappa_1)$ and the dispersion $(\varkappa_2)$ of the logarithm of the breakdown coefficient exist, we introduce the normalized value of the logarithm of the breakdown coefficient and the corresponding characteristic function

$$\zeta_{r,l} = [z_{r,l} - \varkappa_1(l/r)]\,\varkappa_2^{-1/2}(l/r), \quad \chi(t, l/r) = \langle \exp\{it\zeta_{r,l}\}\rangle \tag{3.2}$$

In the same manner as the proof of the integral limiting theorem [27] it may be shown that as $\ln(l/r) \to \infty$

$$\chi(t, l/r) \to \exp\left\{-\frac{1}{2}t^2\right\} \tag{3.3}$$

is uniform in any limiting interval $|t| \leqslant T$ (*). Thus, the breakdown coefficient has a logarithmic-normal probability distribution for $\ln(l/r) \to \infty$.

The quantities related to the ultimate logarithmic-normal distribution will be indicated by a superscript asterisk. From (3.2), (3.3) we have

$$\alpha^*(s) = \alpha^{(1)}(0)\,s + {}^1/_2\alpha^{(2)}(0)\,s^2 \tag{3.4}$$

Substituting (3.4) into (2.9) with (3.1) taken into account, we obtain

$$\mathring{\mu}_p = {}^1/_2 p\,[(p-1)(\mathring{\mu}_2 - 2\mathring{\mu}_1) + 2\mathring{\mu}_1] \tag{3.5}$$

$$2\mu_1^*\ln(l/r) = 2\varkappa_1 + \varkappa_2, \quad \mu_2^*\ln(l/r) = 2(\varkappa_1 + \varkappa_2) \tag{3.6}$$

So far as $\varkappa_2 > 0$, it is evident from (3.6) that $\mu_2^* - 2\mu_1^* > 0$. Thus, (3.5) yields quadratic dependence of the exponent upon the number of the moment. This contradicts the condition (2.5), at least for sufficiently large p. This means that although the distribution of the breakdown coefficient tends toward the logarithmic-normal distribution, the moments do not tend towards the expressions which result from the limiting distribution.

For simplicity let us consider below the case when nonhomogeneous breakdown may

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*) To clarify the proof let it be assumed that function $\alpha(s)$ has all derivatives at zero, and let that function be expanded in series. With (2.8), (3.1) and (3.2) taken into account, we have

$$\chi(t, l/r) = \exp\{-it\varkappa_1\varkappa_2^{-1/2} - \alpha(t\varkappa_2^{-1/2})\ln(l/r)\} =$$

$$= \exp\left\{-\frac{1}{2}t^2 - (\ln(l/r))^{-1/2}\sum_{p=3}^{\infty}\frac{\alpha^{(p)}(0)}{p!}t^p(\alpha^{(2)}(0))^{-1/2p}(\ln(l/r))^{1/2(3-p)}\right\} \to$$

$$\xrightarrow[\ln(l/r)\to\infty]{} \exp\left\{-\frac{1}{2}t^2\right\}$$

For the proof it is really sufficient that the first two moments for the logarithm of the breakdown coefficient exist.

be disregarded. In this case the mean value of the breakdown coefficient equals unity, $\mu_1 = 0$ (2.6). The first of the Eqs. (3.6) yields, in the general case, $\mu_1^* \neq 0$ since for the limiting distribution fulfilment of the condition $2\varkappa_1 + \varkappa_2 = 0$ is not required. Thus, even the mean value of the breakdown coefficient calculated by the limiting logarithmic-normal law, is not required to agree with the true mean value. In analogy in the general case $\mu_2^* \neq \mu_2 \equiv \mu$.

The empirical data presented in Sect. 5 show that the deviation from the logarithmic-normal law begins already with the first order moment. However, the interpretation of these data in terms of the breakdown coefficient is of the nature of an approximation as stated in Sect. 5.

Let us consider the case when the limiting distribution is such that $\mu_1^*=0, \mu_2^* = \mu$. In this case Eq. (3.5) yields

$$\overset{*}{\mu}_p = \frac{1}{2}\mu p(p-1), \qquad \overset{*}{\mu}_{p+1} - \overset{*}{\mu}_p = \mu p \tag{3.7}$$

Taking in account the value $\mu \approx 0.4$ we see that a contradiction with the restriction (2.4) arises at least for $p \geqslant 3$. It is necessary to keep in mind the conditions just pointed out when considering the influences of intermittency upon the structure of the velocity field in the frame of the ideas presented in [5, 6, 21] and also in the formulation of the problem proposed in [7, 9].

In a recent paper [28] an apprehension has been expressed that the known chain of the equations for the moments of the velocity field which results from the equations of hydrodynamics cannot yield a basis for the establishment of a theory of turbulence since the logarithmic-normal distribution is not determined by its moments.

However, the investigation carried out above shows (the necessary restriction (2.5) was obtained in [3]) that with the Friedman-Keller equations everything is in order, although here also arises a peculiar situation. The true distribution is defined uniquely by its moments, but the moments cannot be calculated on the basis of the limiting logarithmic-normal distribution (at least the moments of sufficiently high order).

4. Model. To illustrate what has been said in the preceding Sections and to establish a basis of comparison with experimental data let us consider the simplest example of a model. Let us assume that the probability distribution for the breakdown coefficient corresponding to a change of scale by a factor of two ($l = 2r$) has a constant density

$$W(q, 2) = {}^1/_2\,\theta(q)\,\theta(2-q) \tag{4.1}$$

where $\theta(x)$ is the single-valued function, equal to zero for $x < 0$ and to unity for $x > 0$ (nonhomogeneity of breakdown is not considered).

The scale change by a factor of two is not chosen arbitrarily. The equations of hydrodynamics are quadratically nonlinear and in spectral presentation they contain a convolution of Fourier components of the velocity field. As a result of this type of equation the energy tends to distribute itself across the spectrum in cascade mode with a decrease of scale by two at each step, and in the statistically steady case the supply and dissipation of energy are mutually balanced.

First of all let us introduce for (4.1) quantities μ_p in accordance with definition (2.3)

$$\mu_p = \log_2\left(\frac{1}{2}\int_0^2 q^p dq\right) = p - \log_2(p+1) \tag{4.2}$$

We note at once that this leads to a value $\mu_2 \approx 0.41$ which agrees with the experimental data. Further, taking account of (3.5), (3.6) a simple calculation yields

$$\varkappa_1 = \ln 2 - 1, \quad \varkappa_2 = 1, \quad \mu_1^* \approx 0.28, \quad \mu_2^* = 2$$

$$\mu_p^* = \frac{p}{2 \ln 2} (p + 2 \ln 2 - 2) \qquad (4.3)$$

Thus, in agreement with what has been said at the end of Section 3, the moments of the limiting logarithmic-normal distribution are very far from the true moments (see Table 1).

Let us calculate now the density of the breakdown coefficient distribution $W(q, l/r)$ corresponding to the arbitrary scale ratio l/r. For this we calculate the characteristic function of the logarithm of the breakdown coefficient for $l = 2r$

$$\psi(s, 2) = \frac{1}{2} \int_0^2 \exp\{is \ln q\} \, dq = 2^{is} (1 + is)^{-1}$$

Further, taking account of (2.8) we have

$$\alpha(s) = -is + \log_2 (1 + is)$$

$$\psi(s, l/r) = (1 + is)^{-\log_2 (l/r)} \exp[is \ln (l/r)] \qquad (4.4)$$

We note that the limit transition to the logarithmic-normal law (3.3) is not difficult to carry out directly with the given model without reference to the limit theorem. By calculation of the Fourier transformation of (4.4) we obtain the density distribution of the logarithm of the breakdown coefficient $Q(z, l/r)$ which is connected with the required density distribution for the breakdown coefficient by the obvious relationship $qW(q) = Q(\ln q)$. Finally we obtain

$$W(q, l/r) = \frac{(\ln l/rq)^{\log_2 (l/r) - 1}}{(l/r) \, \Gamma(\log_2 (l/r))} \theta(q) \theta(l/r - q) \qquad (4.5)$$

where $\Gamma(x)$ is the gamma function. The density distribution (4.5) for $l > 2r$ is far from the uniform density $W(0, l/r) = \infty, \quad W(l/r, l/r) = 0$

5. Comparison with experimental data. The available experimental material on intermittency [14–20, 22–25, 29] pertains to the investigation of the quantities of the type (1.1) but not of the breakdown coefficients for which universal laws are to be expected. In the interpretation of these data in terms of the breakdown coefficient, the transition to the limit (1.6) could be utilized, but in doing so, we would transgress the range of similarity of scale. As a consequence the comparison with the theory is informatory in nature. Least sensitive to the extrapolation indicated are evidently such characteristics as the exponents of the moments of various order.

Table 1

origin	[14–16,24]	[17]	[18]	[19]	[20,29]	(4.2)	(4.3)
μ_2	~0.4	0.35±0.07	0.35	0.51, 0.44	0.5	0.41	2
μ_3	—	1.05±0.21	0.93×	—	—	1	5.2
μ_4	—	2.1±0.42	1.68×	—	—	1.68	9.8

Table 1 shows the information available at present in regard to those exponents. The data in [16–20, 29] relate to the first of the quantities (1.1) in [14, 15] to the second, and in [24] to the last(*). For comparison are shown the values (4.2) given in the model investigated above and the corresponding limiting values (4.3) which correspond to the logarithmic-normal law.

As seen from Table 1, the value μ_2 may be regarded as lying within the limits of 0.35–0.5, while the value given by the model 0.41 lies in the middle of that interval. It is noted that for different fields (1.1) the parameters μ_p, generally speaking, may be different.

Higher order moments ($p = 3,4$) of quantities $y_r\ (x)$ (1.3) were determined only in paper [17]. These data are regarded as preliminary since (as stated in [17]) the readings were short (15–20 sec.) and the start of the reading was chosen at the appearance of large signals. Nevertheless paper [17] is the first observation of the fact that the higher order moments are of an exponential nature. Entered into Table 1 is the range of the value μ_p stated in [17] and corresponding to three different readings.

In [18] the quantity μ_2 was determined in the usual manner over the spectrum and, besides, the quantities $\langle y^p \rangle$ for $p = 1, 2, 3, 4$. We have included in the Table the values μ_3 and μ_4 (indicated by a cross) evaluated in the following manner:

$$\mu_p = \mu_2 \frac{\ln a_p}{\ln a_2}\ (p = 3, 4),\ a_p \approx \langle y^p \rangle \langle y \rangle^{-p} \qquad (5.1)$$

The role of r and l in the expression for the moments (2.3) is taken here by the micro- and macro-scales of turbulence. These estimates are close to the values given by the model.

In paper [18] are calculated also the quantities $\langle y^p \rangle$ on the basis of the logarithmic-normal law, very close (as to its mean square value) to the empirical distribution. Substituting these values into the numerator of the second formula of (5.1) we obtain an estimate of the moments of the breakdown coefficient a_p^* calculated on the basis of the logarithmic-normal law

$p =$	1	2	3	4
$\overset{*}{a}_p / a_p =$	1.24	2.05	5.84	35.4

It is seen that these ratios grow with increase of the order of the moment in agreement with what was discussed in Sect. 3. The empirical diagram presented in [18] illustrates the curvature in coordinates in which the logarithmic-normal law appears as a straight line. The same situation also exists in [17, 20, 25, 29] (**).

Some experimenters are inclined to blame the curvature in the range of small values of the argument on the presence of noise in the instrumentation. But one should think that this deviation from the asymptote exists also apart from the noise. It is difficult to imagine that the density distribution of the angular velocity of rotation of a fluid particle vanishes at zero and has a steep maximum, as it follows from the logarithmic-normal law. The more so because the nominal mean value of the angular acceleration for constant angular velocity equals zero (***).

*) The second figure shown in [19] for μ_2 was obtained in an indirect way.
**) In [29] the distribution curve is plotted for the case of moderate Reynolds numbers. For this case the value $\mu_2 \approx 0.85$ is unusually high. The value of μ_2 included in Table 1 obtained from that same paper is for high Reynolds numbers.
***) On the statistical description of a turbulent vortex see the thesis for a Doctor's degree of the author: "Statistical Models in the Theory of Turbulence". M., 1969.

It is noted that the model density distribution (4.5) at zero goes to infinity, which does not interfere with its tending to the logarithmic-normal law (in the integral sense). As far as the region of large values of the argument is concerned, here the limiting distribution necessarily deviates from the logarithmic-normal asymptotics by virtue of the restriction (1.8).

In conclusion let us make corrections for the similarity laws for the structural functions of the velocity field using the model distribution. According to [5, 6, 21]

$$\langle [v_r(\mathbf{x}+\mathbf{r}) - v_r(\mathbf{x})]^p \rangle \sim \langle \varepsilon_r^{1/_3 p} \rangle r^{1/_3 p} \sim r^{1/_3 p - \mu_{1/_3 p}}$$

where the index r denotes the velocity component in the $\mathbf{r}$ direction. Equation (4.2) gives specifically

$$\mu_{2/_3} \approx -0.07, \qquad \mu_{4/_3} \approx 0.11 \tag{5.2}$$

These values may be compared with those which are obtained for the logarithmic-normal distribution (3.7) (if it is formally assumed that $\mu_1^* = 0$, $\mu_2^* = \mu$)

$$\mu_{2/_3} = -{}^1/_9\mu, \; \mu_{4/_3} = {}^2/_9\mu \tag{5.3}$$

The first of the estimates (5.3) was made in [21], where for $\mu = 0.4$ the value $\mu_{2/_3}$ ≈ -0.04 was obtained. Corrections ${}^2/_3$ for the law turn out to be low in both cases. In [20] are given the results of measurements of the structural function of fourth order and a correction is found which agrees with (5.3) (for the value $\mu \approx 0.5$) obtained in [20]) as well as with (5.2).

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